RELATION BETWEEN THE HYDRODYNAMIC PARAMETERS IN ZONES OF LOCAL DISTURBANCES OF THE FLOW

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The conservation laws of hydrodynamics in integral form, in the general case of internal flows with the usual assumptions for quasi-one-dimensional approximations, lead to an open system of equations. In different applications these equations are traditionally solved using additional and often contradictory hypotheses [1-4] or experimental data [5]. In [2, 6], where the flow of an ideal gas in channels with steps is studied, it is noted that the pressure at a step in the channel is related with the jump in the entropy. Here, in order to close the equations of hydrodynamics, the hypothesis that the thermodynamic function is independent is proposed — the coefficient of restoration of the pressure σ_p is independent of the jump in the cross section of the channel. The dependence could be manifested only implicitly when dissipative effects, arising in the zones of mixing with different types of local perturbations of the flow, are taken into account. This hypothesis is based on a qualitative analysis of the first and second laws of thermodynamics, and in its turn permits constructing an additional equation that closes the system of equations of hydrodynamics.

1. The laws of conservation of hydrodynamics are written down for a fixed volume V of part of the channel, within which the action on the liquid occurs and new hydrodynamic parameters of the flow are established (Fig. 1). The quasi-one-dimensional treatment of the flow in the channel and in the side branches and under typical assumptions for such problems [7, 8] leads to the following system of equations for the average parameters:

$$\begin{gathered}
(S\rho v)^+ = (S\rho v)^- - \sum_{k=1}^{m} (\varepsilon S\rho v_n)_k, \quad (\alpha S\rho v\mathbf{v} + S\mathbf{n}p)^+ - \\
- (\alpha S\rho v\mathbf{v} - S\mathbf{n}p)^- + p_\sigma \int\limits_{\sigma+\Sigma S_k^-} \mathbf{n}\,dS - \int\limits_{\sigma_t} \boldsymbol{\tau}_n^* dS - \int\limits_{S^- + S^+} \mathbf{P}_n dS + \\
+ \sum_{k=1}^{m} \left\{ (\alpha S\rho v_n^2)_k \int\limits_{(\varepsilon S)_k} \mathbf{n}\,dS + p_k \int\limits_{S_k} \mathbf{n}\,dS - p_\sigma \int\limits_{S_k^-} \mathbf{n}\,dS - \int\limits_{\sigma_{t^k}} \boldsymbol{\tau}_n^* dS - \int\limits_{S_k} \mathbf{P}_n dS \right\} = 0, \qquad (1.1) \\
(S\rho vH)^+ - (S\rho vH)^- + \int\limits_{\sigma_t} q_n^* dS + \int\limits_{S^- + S^+} Q_n dS + \sum_{k=1}^{m} \left\{ (\varepsilon S\rho v_n H)_k + \int\limits_{\sigma_{t^k}} q_n^* dS + \int\limits_{S_k} Q_n dS \right\} = 0.
\end{gathered}$$

Here p, ρ, and v are the pressure, density, and velocity; $H = \beta v^2/2 + \gamma p/(\gamma - 1)\rho$; α and β take into account the nonuniformity of the starting velocity field; ε is the compression ratio of the flow; **n** is a normal vector; $\boldsymbol{\tau}_n^* = \boldsymbol{\tau}_n + \mathbf{P}_n$, $\mathbf{P}_n = -\rho\langle v_n' \mathbf{v}' \rangle$, $\mathbf{q}^* = \mathbf{q} + \mathbf{Q}$, $\mathbf{Q} = \rho \langle \mathbf{v}' v'^2 \rangle / 2$, and $\boldsymbol{\tau}_n$, **q** are the friction stress vector and the heat flux vector; $\mathbf{P}_n$ and **Q** are the same for turbulent pulsations; the indices − and + indicate a state in the starting S^- and output S^+ sections of the channel; k indicates the cross section of the k-th side branch; and, σ denotes parameters evaluated on the side surface of the channel. The equations (1.1) were

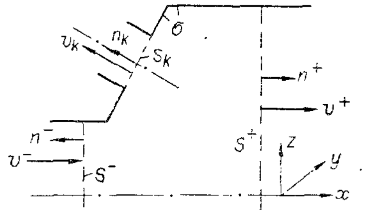

Fig. 1

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written out under the assumption that the axes of the side channels lie in the symmetry planes of the channel and are orthogonal to its side surface. Relation of the type $\int_{\sigma+\Sigma\sigma_k} \boldsymbol{\tau}_n dS = \int_{\sigma_t} \boldsymbol{\tau}_n dS + \sum_{k=1}^{m} \int_{\sigma_{tk}} \boldsymbol{\tau}_n dS$ hold for the friction forces and heat fluxes on the surface of the channel σ and the side branches σ_k (σ_t is he surface of the screen tube of the flow of gas from the section S^- into S^+, σ_{tk} is the same for the flow crossing the section S_k). Some types of work performed in the volume V, for example, by turbulent stresses, are omitted in the energy euqation. In the solution presented here the latter are important only as a physical factor due to the presence of turbulent pulsations, already mentioned in the energy equation. The equations (1.1) are supplemented by the equation of state $p = \rho RT$ (T is the temperature).

Taking into account the facts that $\int_{\sigma+\Sigma S_k^-} \mathbf{n} dS = -(S^+ - S^-)\,\mathbf{i}$, $\int_{S_k} \mathbf{n} dS = \int_{S_k^-} \mathbf{n} dS = S_k (n_{kx}\mathbf{i} + n_{kr} \times (r_{ky}\mathbf{j} + r_{kz}\mathbf{k}))$ ($\mathbf{n}_k$ is a unit vector on the axis of the k-th side branch and $\mathbf{r}_k$ is a unit vector lying in the plane ($\mathbf{n}_k$, $\mathbf{i}$) and is orthogonal to the axis of the channel) the projections of the equations of motion (1.1) on the transverse coordintes (see Fig. 1) lead to

$$p_k + (\varepsilon\rho v_n^2)_k (\alpha_k + \Delta_{\tau k}/n_{kr}) = p_\sigma \quad (k = 1, m), \tag{1.2}$$

where $\Delta_{tk} = -\left(\int_{\sigma_t^k} \tau_{nr}^* dS + \int_{\sigma_{tk}} \tau_{nr} dS + \int_{(S^-+S^+)^k + S_k} P_{nr} dS\right) \Big/ (\varepsilon S\rho v_n^2)_k$ determines the role of the transverse components of the friction acting on the k-th stream tube; σ_t^k is the part of the surface of the stream tube σ_t adjacent to the stream tube σ_{tk}; $(S^- + S^+)^k$are the transverse cross sections of the stream tube σ_{tk}; in the sections S^- or S^+.

In connection with the energy equation we note a difference in (1.1) for outflow from the channel and inflow into the channel. In the first case it can be transformed taking into account the energy equations for small streams crossing the section S_k: $(\varepsilon S\rho v_n)_k H_k + \int_{\sigma_{tk}} q_n^* dS + \int_{S_k} Q_n dS = (\varepsilon S\rho v_n)_k H^-$; in the second case the enthalpy H_k is an external (given) parameter. Introducing the weighing factor φ_p for the average pressure $p_c = \varphi_p p^+ + (1 - \varphi_p)p^-$, the notation

$$\Delta_\tau = -\left(\int_{\sigma_t} \tau_{nx}^* dS + \int_{S^-+S^+} P_{nx} dS + \sum_{k=1}^{m}\left(\int_{\sigma_{tk}} \tau_{nx} dS + \int_{S_k} P_{nx} dS\right)\right) \Big/ G^- v^-,$$

$$\Delta_Q = -\left(\int_{\sigma_t} q_n^* dS + \int_{S^-+S^+} Q_n dS\right) \Big/ (Ga^2)^-, \quad \Delta_{Qk} =$$

$$= -\left(\int_{\sigma_{tk}} q_n^* dS + \int_{S_k} Q_n dS\right) \Big/ G_k (a^-)^2, \tag{1.3}$$

$$\Delta_S = 1 - S^-/S^+, \ G = (S\rho v)^-, \ G_k = (\varepsilon S\rho v_n)_k, \ \Delta_{Gk} = -G_k/G^-,$$

$$\Delta_G = \sum_{k=1}^{m} \Delta_{Gk}, \quad a^2 = \gamma p/\rho, \quad \Delta_H = \Delta_Q + \sum_{k=1}^{m} \Delta_{Gk} \left(A_{Hk} (H/a^2)^- - \theta_k \Delta_{Qk}\right)$$

($A_{Hk} = 1$, $\theta_k = 0$ for outflow from the channel, and $A_{Hk} = H_k/H^-$, $\theta_k = 1$ for inflow into the channel), and eliminating the pressure p_k with the help of (1.2), Eqs. (1.1) reduce to (1.2) and the conditions

$$(S\rho v)^+ = (1 + \Delta_G)(S\rho v)^-, \ (1 + \Delta_G)H^+ = H^- + \Delta_H (a^-)^2, \tag{1.4}$$

$$(\alpha S\rho v^2 + \sigma p)^+ = (\alpha S\rho v^2 + \sigma p)^- - (Gv)^- \Delta_\tau^*;$$

$$\Delta_\tau^* = \Delta_\tau + \sum_{k=1}^{m} (v_{nk}/v^-) (\Delta_G \Delta_\tau n_x/n_r)_k, \quad \sigma^+ = S^+ (1 - \varphi_p \Delta_S), \quad \sigma^- = \sigma^+. \tag{1.5}$$

The system (1.4) describes the reaction of gas to local finite disturbances: geometric Δ_S, flow Δ_{Gk}, thermal Δ_Q and Δ_{Qk}, and friction forces Δ_τ and $\Delta_{\tau k}$. For given disturbances the system (1.4) must be studied independently of (1.2), and in the general case it is open owing to the fact that p_σ (or φ_p) is unknown. The equations (1.2) in which the external and internal (with respect to the volume of liquid singled out) parameters of state, together with the energy equations for stream tubes of the side branches and other external conditions, can be employed to determine some of the disturbances enumerated above. In particular, in problems in which mass is supplied to the channel ($\Delta_{Gk} > 0$) it is usually assumed that $p_k = p_\sigma$ [9]. Then (1.2) give fully determined values $\Delta_{\tau k} -\alpha_k n_k$, and (1.5) leads to.

$$\Delta_\tau^* = \Delta_\tau - \sum_{k=1}^{m} J_{kx}/(S\rho v^2)^-, \quad J_{kx} = -\left(\alpha\varepsilon S\rho v_n^2 n_x\right)_k. \tag{1.6}$$

Here J_{kx} is the projection of the momentum of the liquid mass, injected into the channel through the k-th opening in the direction of the flow in the channel. With (1.6) the system (1.4) is identical, under comparable conditions ($\Delta_G > 0$, $|\Delta_S| \ll 1$), to the equations of [9], and Δ_τ^* is the effective coefficient of friction introduced in [10].

Unlike the problems presented above, problems with removal of mass ($\Delta_{Gk} < 0$), where $p_k \neq p_\sigma$, can be studied on the basis of the model of an ideal liquid ($\Delta_{\tau k} = \Delta_\tau = \Delta_\tau^* = Q_n = 0$), which makes Eqs. (1.4) "insensitive" to the angle at which mass is removed from the channel.

2. The solution of the system (1.4) relative to the parameters of the state after the disturbances gives a quadratic equation, in whose coefficients it is convenient to replace the ratio of the heat capacities γ by an effective quantity n (under the additional restrictions $\alpha^+ = \beta^+ = 1$):

$$n = (1 - (\gamma - 1)\sigma^+/\gamma S^+)^{-1} = \gamma/(1 + (\gamma - 1)\varphi_p\Delta_S). \tag{2.1}$$

The quadratic equation mentioned, together with the equation of state, leads to the relations

$$\begin{gathered}
\frac{v^+}{v^-} = \frac{K \pm \nu N}{(n+1)\,\mathrm{M}^{-2}(1+\Delta_G)}, \quad \frac{\rho^+}{\rho^-} = \frac{(1-\Delta_S)(n+1)\,\mathrm{M}^{-2}(1+\Delta_G)^2}{K \pm \nu N}, \\
\frac{p^+}{p^-} = \frac{K \mp \nu n N}{I(n+1)}, \quad \frac{T^+}{T^-} = \frac{(K \mp \nu n N)(K \pm \nu N)}{I(1-\Delta_S)(n+1)^2\,\mathrm{M}^{-2}(1+\Delta_G)^2},
\end{gathered} \tag{2.2}$$

$$\begin{gathered}
\mathrm{M}^{+2} = (1-\Delta_S)\,I\,\frac{K \pm \nu N}{K \mp \nu n N}, \quad \frac{p_\sigma}{p^-} = 1 + \varphi_p\left(\frac{p^+}{p^-} - 1\right); \\
\mathrm{M} = v/a, \ \mathrm{M}^{-2} = (\mathrm{M}^-)^2, \ I = n\sigma^-/\gamma S^-, \ \nu = \mathrm{sign}(\mathrm{M}^- - 1), \\
K = I + n\mathrm{M}^{-2}(\alpha^- - \Delta_\tau^*), \quad N = \{K^2 - 2(n^2-1)\,\mathrm{M}^{-2}(c_0 + \\
+ (\alpha^-)^2(\gamma-1)\,\mathrm{M}^{-2}/2)/(\gamma-1)\}^{1/2}, \\
c_0 = c_1 + (\gamma-1)\,\mathrm{M}^{-2}c_2/2, \quad c_2 = \beta^-(1+\Delta_G) - (\alpha^-)^2 + (1+\Delta_G)\sum_{k=1}^{m} A_{Hk}\Delta_{Gk}, \\
c_1 = (1+\Delta_G)\left(1 + \sum_{k=1}^{m} A_{Hk}\Delta_{Gk} + (\gamma-1)\left(\Delta_Q - \sum_{k=1}^{m}\theta_k\Delta_{Qk}\right)\right).
\end{gathered} \tag{2.3}$$

The relations (2.2), when there are no disturbances and the upper sign is chosen, give a trivial result, while the relations with the lower sign give the Hugoniot–Rankine relation on a straight shock. When disturbances are present the reaction of the gas is different in the region of sub- and supersonic flows [9]; this is taken into account by introducing the function ν into the solution. Denoting

$$X = \{x_i\} = \{\mathrm{M}^-, \Delta_{Gk}, \Delta_Q, \Delta_{Qk}, \Delta_\tau, \Delta_{\tau k}\}, \tag{2.4}$$

we find from the relations (2.2) and (2.3) that the general structure of the solution for the hydrodynamic parameter ψ has the form

$$\psi^+/\psi^- = f[X, \Delta_S, \varphi_p(X, \Delta_S)], \tag{2.5}$$

where the unknown quantity φ_p is a function of all disturbances enumerated above.

3. For problems studied here the analysis of the first and second laws of thermodynamics permits formulating a more general hypothesis, which enables closing the system of equations of hydrodynamics. The relations mentioned, under the assumptions employed above (see Sec. 1) and representing the enthalpy in the energy equation (1.4) in the form

$$H = (\gamma p_H^{1/\gamma}/(\gamma-1)\rho_H)\, p_0^{(\gamma-1)/\gamma} \exp\left(\frac{s-s_H}{c_p}\right)$$

can be written as follows [11]:

$$s^+ - s^- = \frac{1}{1+\Delta_G}\left\{\frac{1}{G^-}\left[\int\limits_{\sigma+\Sigma\sigma_k}(q_n/T)\,dS + \int\limits_V\left(\frac{D}{T}+\varkappa\left(\frac{\nabla T}{T}\right)^2 + (\nabla\cdot(P\cdot v) - \nabla\cdot Q)/T\right)dV\right] + \sum_{k=1}^{m}(s_k/s^- - 1)\,\Delta_{Gk}\right\}; \tag{3.1}$$

$$\sigma_p = p_0^+/p_0^- = \left[(1+(\gamma-1)\tau(\mathrm{M}^-)\Delta_H)\exp((s^- - s^+)/c_p)/(1+\Delta_G)\right]^{\gamma/(\gamma-1)}, \tag{3.2}$$

where $\tau(M) = (1 + (\gamma - 1)M^2/2)^{-1}$; s is the entropy; D is the dissipative function; $\varkappa$ is the thermal conductivity; P is the turbulent-stress tensor; p_0 is the stagnation pressure; and, the "initial" states of the medium, determined by the parameters p_H and ρ_H, in the sections S^- and S^+ are assumed to be identical. According to (3.1) and (3.2) the stagnation pressure in the zone of restructuring of the flow can vary owing to the following: inflow of heat and mass (with one or the other sign); dissipative processes owing to the viscosity of the medium (the first two terms in the volume integral in (3.1)); and, nonuniformities of the fields of the turbulent stresses and heat flux. All reasons enumerated above for the change in the mechanical energy of the flow, characterized by the coefficient of restoration of the pressure σ_p, were previously represented as dimensionless parameters – the actions. Thus the first and second laws of thermodynamics, using the notations (2.4), lead to the conclusion that there exists a thermodynamic function

$$\sigma_p^{(1)} = f_1(X), \tag{3.3}$$

which is what served as a basis for formulation of the hypothesis stated above. In the general case the function $f_1(X)$ is unknown, and does not give any quantitative information.

On the other hand, the relation (2.2), which follow from the three conservation laws of hydrodynamics, also permit constructing the ratio p_0^+/p_0^- as a known function of the actions and the weighting coefficient of the average pressure φ_p:

$$\sigma_p^{(2)} = \frac{\pi(\mathrm{M}^-)}{I(n+1)}(K \mp \nu n N)\left(1 - \frac{n-1}{2}\frac{K \pm \nu N}{K \mp \nu n N}\right)^{\gamma/(\gamma-1)} \tag{3.4}$$

$(\pi(\mathrm{M}) = (\tau(\mathrm{M}))^{\gamma/(\gamma-1)})$, whence

$$\sigma_p^{(2)} = f_2(X, \Delta_S, \varphi_p(X, \Delta_S)). \tag{3.5}$$

The dependence of $\sigma_p^{(2)}$ on the arguments listed in (2.5) is not single-valued, as are the relations (2.2) or (2.5), from which it is obtained owing to the uncertainty of the function $\varphi_p(X, \Delta_S)$.

In the relations (3.3) and (3.5) the actions X(2.4) and DS can be regarded as independent variables. But unlike other actions, the actions Δ_τ and $\Delta_{\tau k}$ can be represented as functions of other actions and some variable parameters λ_i (for example, $\Delta_\tau(\Delta_S, \Delta_Q, \Delta_{Qk}, \Delta_{Sk}, \mathrm{M}^-, \lambda_i)$). This is explained, on the one hand, by the fact that any actions that alter the starting state of the flow are accompanied by dissipative effects and change Δ_τ. Moreover, even when the actions $\Delta_S, \Delta_Q, \ldots, \mathrm{M}^-$ are fixed, the value of Δ_τ can vary over a wide range owing to λ_i (different shape of the steps in the channel, the presence of chambers, nets across the channel, different degree of roughness of the side walls or starting intensity of the turbulence, etc.); this is what justifies the starting assumption. Analogous arguments also hold for $\Delta_{\tau k}$, and with some modifications for Δ_Q, Δ_{Qk} also.

In the expressions (3.2) and (3.4) one and the same thermodynamic function is defined; this enables writing for $\sigma_p^{(1)}$ and $\sigma_p^{(2)}$ the condition

$$\sigma_p^{(1)} = \sigma_p^{(2)} \text{ or } d\sigma_p^{(1)} - d\sigma_p^{(2)} = 0, \tag{3.6}$$

which, together with the relations (2.4), (3.3), and (3.5), gives

$$\left[\frac{\partial f_1}{\partial x_i}-\left(\frac{\partial f_2}{\partial x_i}+\frac{\partial f_2}{\partial \varphi_p}\frac{\partial \varphi_p}{\partial x_i}\right)\right]dx_i+\left[-\left(\frac{\partial f_2}{\partial \Delta_S}+\frac{\partial f_2}{\partial \varphi_p}\frac{\partial \varphi_p}{\partial \Delta_S}\right)\right]d\Delta_S=0.$$

Because the variables x_i and Δ_S are independent, from here follow

$$\frac{\partial f_1}{\partial x_i}-\left(\frac{\partial f_2}{\partial x_i}+\frac{\partial f_2}{\partial \varphi_p}\frac{\partial \varphi_p}{\partial x_i}\right)=0,\quad \frac{\partial f_2}{\partial \Delta_S}+\frac{\partial f_2}{\partial \varphi_p}\frac{\partial \varphi_p}{\partial \Delta_S}=0. \tag{3.7}$$

The relations (3.7) ensure that the conditions (3.6) are satisfied, and the latter condition

$$\frac{\partial \varphi_p}{\partial \Delta_S}=-\frac{\partial f_2}{\partial \Delta_S}\left(\frac{\partial f_2}{\partial \varphi_p}\right)^{-1}=F(\Delta_S,\varphi_p,X)$$

ensures that the function $\sigma_p^{(2)}$ is independent of the geometric action Δ_S as an independent variable. Such a dependence can occur only implicitly through the actions Δ_τ, $\Delta_{\tau k}$, and Δ_Q, Δ_{Qk} and is taken into account in (3.4). Equation (3.8) together with (2.2) describes the reaction of the gas in the channel to local finite actions Δ_S, Δ_{Gk}, Δ_Q, Δ_{Qk}, Δ_τ, $\Delta_{\tau k}$, forming in the process a closed system.

4. We shall study the behavior of the gas under local disturbances first for the example of an incompressible liquid ($M^-\ll 1$, $M^+\ll 1$). Expanding the functions contained in (2.2) and (3.4) in a series in powers of the Mach number and retaining the terms no higher than quadratic in M ($N\simeq K-(n+1)M^{-2}(1-\Delta_S)c_1$), we obtain

$$v^+/v^-=(1-\Delta_S)(1+\Delta_G),\ p_\sigma/p^-=1+(c_1\Delta_S+2\delta)/2\mathrm{Eu}^-,\ \rho^+/\rho^-=1,$$
$$p^+/p^-=1+(c_1\Delta_S(2-\Delta_S)+2\delta)/2\mathrm{Eu}^-,\ T^+/T^-=1,\ \delta=\alpha^--c_1-\Delta_\tau^*,$$
$$c_1=(1+\Delta_G)^2,\ \mathrm{Eu}^-=(p/\rho v^2)^-,\ \Delta_\tau^*=\Delta_\tau-\sum_{k=1}^{m}\Delta_{Gk}^2\Delta_{\tau k}(n_x/\varepsilon S^0 n_r)_k,$$

where $S_k^0=S_k/S^-$ and $\Delta_{\tau k}=-\alpha_k n_{kr}$ with mass inflow (see Sec. 1);

$$\sigma_p^{(2)}=\frac{2\mathrm{Eu}^-}{1+2\mathrm{Eu}^-}\frac{1}{I(n+1)}\left(K+nN+\frac{\gamma}{2}\frac{n-1}{\gamma-1}(K-N)\right). \tag{4.1}$$

The condition (3.8), in application to (4.1), leads to a differential equation with the parameters Δ_{Gk}, Δ_τ, $\Delta_{\tau k}$:

$$\Delta_S(1-\Delta_S)(\delta+\Delta_S c_1)(d\varphi_p/d\Delta_S)=(1-\varphi_p)(\delta+\Delta_S c_1)-$$
$$-\varphi_p\Delta_S(1-\Delta_S)(1-\varphi_p\Delta_S)c_1,$$

whose solution has the form

$$\varphi_p=\left(c_1\Delta_S^2-2\delta(1-\Delta_S)+2f\right)/\left(c_1\Delta_S^2(2-\Delta_S)+2\Delta_S f\right).$$

The function of the parameters f can be determined using one of the relations (2.1) or (2.3) for n or I, related with φ_p, whose values at the point $\Delta_S = 0$ are known: $n = \gamma$ and $I = 1$. Either one gives $f = \delta$, and leads to the final expressions for the functions φ_p and σ_p and the hydrodynamic parameters:

$$\varphi_p=\frac{c_1\Delta_S+2\delta}{c_1\Delta_S(2-\Delta_S)+2\delta},\quad \sigma_p=\frac{c_1+2\mathrm{Eu}^-+2\delta}{1+2\mathrm{Eu}^-},$$
$$v^+/v^-=(1-\Delta_S)(1+\Delta_G),\ p_\sigma/p^-=1+(c_1\Delta_S+2\delta)/2\mathrm{Eu}^-,$$
$$p^+/p^-=1+(c_1\Delta_S(2-\Delta_S)+2\delta)/2\mathrm{Eu}^-. \tag{4.2}$$

Applying (4.2) for describing the flow of an ideal incompressible liquid in a channel with impermeable walls ($c_1 = 1$, $\delta = 0$) gives

$$\varphi_p=1/(2-\Delta_S),\ \sigma_p=1. \tag{4.3}$$

The last of the relations (4.2) is a combination of the Bernoulli integral with the equation of continuity.

To describe the flow of a real liquid in a channel based on the relations (4.2) it is necessary to determine additionally Δ_τ and $\Delta_{\tau k}$ (with the exception of mass inflow, when

$\Delta_{\tau k} = -\alpha_k n_{kr}$). In many cases the experimental hydraulics data on the coefficients of the hydraulic losses ξ can be employed for this; the relation between these coefficients and the coefficient of restoration of the pressure $\xi = (1 + 2\mathrm{Eu}^-)(1 - \sigma_p)$ gives

$$\xi = 1 - c_1 - 2\delta = 2(1 - \alpha^-) + \Delta_G(2 + \Delta_G) + 2\Delta_\tau^*.$$

Here the hydraulic losses are referred to the velocity v^-. As an illustration we shall examine the well-known problem of Borda about the flow of liquid in a channel with a sudden expansion of the cross section ($\Delta_S > 0$, $\Delta_{Gk} = 0, \Delta_\tau \neq 0$)., for which experimental data on the hydraulic losses are satisfactorily described by the expression $\xi = \Delta_S^2$ [12] and, together with (4.2), lead to the relations

$$\Delta_\tau = \Delta_S^2/2,\ \varphi_p = 1/2,\ v^+/v^- = 1 - \Delta_S,$$
$$p^+/p^- = 1 + \Delta_S(1 - \Delta_S)/\mathrm{Eu}^-,\ p_\sigma/p^- = 1 + \Delta_S(1 - \Delta_S)/2\mathrm{Eu}^-. \tag{4.4}$$

The result obtained here with regard to $\varphi_p = 1/2$ ($p_\sigma = (p^+ + p^-)/2$) does not agree with the hypothesis $p_\sigma = p^-$ ($\varphi_p = 0$) widely employed in hydraulics [13] and must be checked experimentally (see Sec. 6).

For flows in channels without flow disturbances, characteristically $p_\sigma \in [p^-, p^+]$ and $0 \leqslant \varphi_p \leqslant 1$ (4.3), (4.4), whereas in problems with flow disturbances the values of p_σ and φ_p fall outside these intervals. In particular, when different actions compensate one another and $p^+ = p^-$, the function $\varphi_p = (p_\sigma - p^-)/(p^+ - p^-)$ has a discontinuity of second order, in the vicinity of which the relation between the pressures p_σ, p^-, and p^+ changes. The combination of actions for which these effects occur can be found from the condition $c_1\Delta_S(2 - \Delta_S) + 2\delta = 0$, following from (4.2) with $p^+ = p^-$.

5. For a compressible liquid Eq. (3.8) can be solved only numerically. But the value of φ_p at the point $\Delta_S = 0$ is unknown, which makes it difficult to formulate the Cauchy problem for this function. Under these conditions it is convenient to reformulate (3.8) for n, related with φ_p by the relation (2.1) and assuming the value $n = \gamma$ at the point $\Delta_S = 0$. As a result

$$a(dn/d\Delta_S) = -b,\ \Delta_S = 0,\ n = \gamma,$$
$$a = [(n - \gamma)(K \mp \nu n N) \pm \nu(n^2 - 1)N]\,[(\gamma - 1)(1 - \Delta_S)\,(K \mp \nu n N) - (n^2 - 1)],$$
$$b = \frac{n(n^2 - 1)}{1 - \Delta_S}\{\pm\nu(\gamma - 1)^2(1 - \Delta_S)\,N(K \mp \nu n N) - (n - 1)[(n - \gamma)(K \mp \nu n N) \pm \nu(n^2 - 1)N]\}. \tag{5.1}$$

The nonlinear differential equation (5.1 has at the point $\Delta_S = 0$ a singularity of the type $(dn/d\Delta_S)_{\Delta_S=0} = 0/0$. To eliminate this singularity the coefficients a and b in (5.1) must be expanded in series in powers of the small quantities $\delta\Delta_S$ and δn about the point $\Delta_S = 0$, $n = \gamma$, and for transsonic flows additionally in powers of $\delta M^- = 1 - M^-$. Retaining the linear terms in the expansion we arrive at a number of particular cases:

$$\frac{dn}{d\Delta_S} = \begin{cases} -\gamma(\gamma - 1):\ |\Delta_S| \leqslant \varepsilon_\Delta,\ \Psi \neq 0, \\ -\gamma(\gamma - 1)/2:\ |\Delta_S| \leqslant \varepsilon_\Delta,\ \Psi = 0,\ |M^- - 1| > \varepsilon_M, \\ -2\gamma(\gamma - 1)/3:\ 0 \leqslant \Delta_S \leqslant \varepsilon_\Delta,\ \Psi = 0,\ |M^- - 1| \leqslant \varepsilon_M, \end{cases} \tag{5.2}$$

where $\Psi = K_0 \mp \nu\gamma N_0 - (\gamma - 1)$; $K_0 = 1 + \gamma M^{-2}(1 - \Delta_\tau^*)$; $N_0 = [K_0^2 - 2(\gamma + 1)M^{-2}(c_0 + (\alpha^-)^2(\gamma - 1)M^{-2}/2)]^{1/2}$; the function Ψ differs from zero, if the ratio of the pressures p^+/p^- at the point $\Delta_S = 0$ differs from unity owing to some actions, otherwise $\Psi = 0$; ε_Δ and ε_M are small in the vicinity of the singular points.

Integration of (5.2) in the neighborhood of $\Delta_S = 0$ and (5.1) outside this neighborhood and using (2.1) give

$$\varphi_p = \{1/(1 - (\gamma - 1)\Delta_S),\ 1/(2 - (\gamma - 1)\Delta_S),\ 2/(3 - 2(\gamma - 1)\Delta_S),\ (\gamma - n)/(\gamma - 1)n\Delta_S\}. \tag{5.3}$$

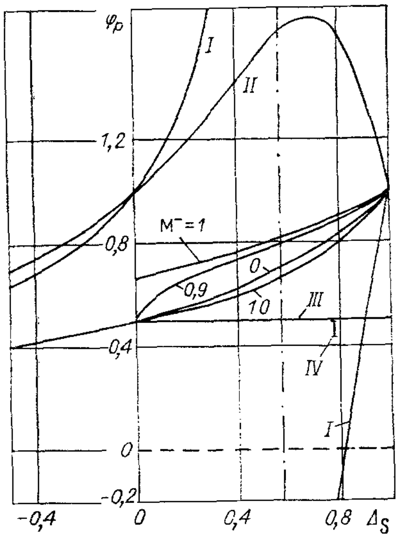

Fig. 2

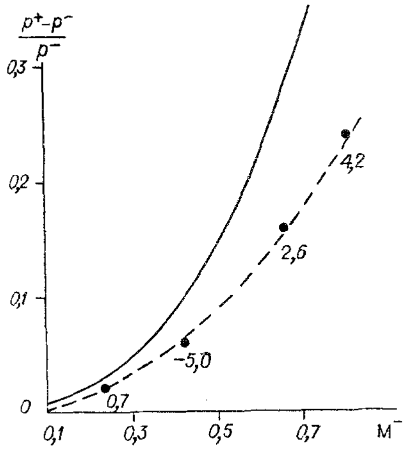

Fig. 3

Here the first three values correspond to (5.2) and the last value corresponds to Eq. (5.1), in which the initial conditions are connected taking into account the solution for n in the neighborhood of $\Delta_S = 0$. For the function n, like for φ_p, different singular points of second order can appear. This is obvious from the relation (2.1), where the vanishing of the denominator gives finite values for the function φ_p for $\Delta_S \neq 0$. From here follows a method for overcoming the singular points in the numerical solution, consistig of transferring from integration of the equation for n to the equation for φ_p with the help of (2.1).

The results of the calculation of φ_p for adiabatic flows of an ideal compressible liquid $(\Delta_\tau = \Delta_{\tau k} = \Delta_{Gk} = \Delta_Q = \Delta_{Qk} = 0)$ in channels with steps are presented in Fig. 2, where the curve with $M^- = 0$ corresponds to the formula (4.3). As expected the values of φ_p under these conditions give ratios of the hydrodynamic parameters that correspond to isentropic flow of a gas at a jump in the cross-sectional are: $p/p^- = \pi(M^+)/\pi(M^-)$, etc. The equivalence of isentropic flows in channels with a step and with a continuous change in the cross-sectional area was employed in [14] by replacing the equation of motion (1.4) with the unknown pressure on the step by a Poisson adiabat. The curves I and II for $\Delta_G = 0.3$ and 0.6 correspond to the formula (4.2), while the curve III corresponds to the formula (4.4); in addition, $\Delta_\tau = 0$ and mass is supplied along the normal to the axis of the channel; the broken line corresponds to Borda's hypothesis $\varphi_p = 0$.

6. To check experimentally some of the results obtained above we shall confine our attention to subsonic and incompressible flows of gas in a channel with a sudden expansion. To describe such flows adequately the dissipative effects in the mixing zones must be taken into account [1]. The latter can be described in both cases with adequate accuracy on the basis of the model of an incompressible liquid [15], where $\Delta_\tau = \Delta_S^2/2$ (4.4). The computed (5.1) and (2.2) values of the pressures in the wide part of the channel are compared with the experimental values [16] for subsonic air flows in the channel with $\Delta_S = 0.498$ in Fig. 3 (the broken line and the dots, respectively), where the numbers on the curve indicate the relative error in the experimental and computed data while the solid curve corresponds to the model of an ideal liquid. For an incompressible liquid the result $\varphi_p = 1/2$ in (4.4) was checked experimentally. The experimental setup practically reproduced the conditions of the experiment of [16]: the pressure distribution was determined by draining the surface of the step and the wide part of the channel. In this case $\Delta_S = 0.77$, $M^- \leqslant 0.1$, $\gamma = 1.4$; the recording and measuring apparatus gave a measurement error of not greater than 0.1 Pa. The experimental value $\varphi_p = (p_\sigma - p^-)/(p^+ - p^-) = 0.470(1 \pm 0.049)$ (the line IV in Fig. 2) with a confidence probability greater than 0.95, and confirm with a satisfactory error the value $\varphi_p = 1/2$ $(p_\sigma = (p^+ + p^-)/2)$.

7. As an illustration of the new possibilities of the solutions (2.2) and (5.1) we shall study the problem of adiabatic efflux of a subsonic flow of ideal liquid from a channel through side branches which are orthogonal to the side surface of the channel. In each side branch the flow is detached from the walls and a pressure equal to the pressure in the surrounding medium p_a is established in some minimum section of the stream tube S_m and on the

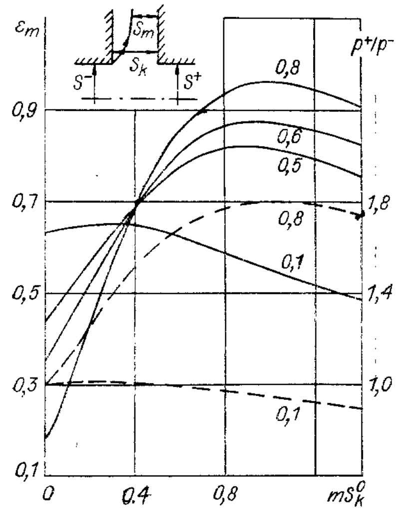

Fig. 4

free surface of the jet. The equations of hydrodynamics (1.2) and (2.2) for p_σ are supplemented by the conservation laws for the liquid between the sections S_k and S_m (diagram in Fig. 4) and lead to the relations

$$(\varepsilon\rho v^2)_m + p_m = p_\sigma,\ p_\sigma/p^- = 1 + \varphi_p\,[(K + nN)/I(n+1) - 1],$$
$$p_m/p_0^- = \tau_m^{\gamma/(\gamma-1)},\ \rho_m/\rho_0^- = \tau_m^{1/(\gamma-1)},\ \tau_m = \tau\,(\mathrm{M}_m),\ p_a = p_m \tag{7.1}$$

($\varepsilon_m = S_m/S_k$ is the coefficient of narrowing of the jet in the side branch). From the definition of Δ_G in (1.3) and ε_m we obtain the following relation

$$\Delta_G = m\Delta_{Gk} = -\,mS_k^0\mu\varepsilon_m,\ \mu = (\mathrm{M}_m/M^-)\,(\tau_m/\tau^-)^{(\gamma+1)/2(\gamma-1)} \tag{7.2}$$

($S_k^0 = S_k/S^-$). The problem of finding the flow rates Δ_G or the coefficients ε_m, formulated above based on the laws of conservation of hydrodynamics, leads to an infinite system of equations owing to the uncertain function $\varphi_p(\Delta_S,\ \Delta_{Gk})$. Using the additional equations (5.1)-(5.3) closes the system (7.1) and (7.2).

For the particular case $\Delta_S = 0$ (channel with a constant cross section), when $n = \gamma$, $I = 1$, $\varphi_p = 1$, $p_\sigma = p^+$, the problem reduces to the solution of a system of nonlinear equations leading to a quadratic equation for Δ_G or ε_m. Its solution gives

$$\Delta_G = m\Delta_{Gk} = mS_k^0\,(A/B - \sqrt{(A/B)^2 + C/B}),$$
$$A = \gamma(\gamma+1)\{\mathrm{M}^-\mathrm{M}_m\,(\tau_m/\tau^-)^{1/2}\,[(\gamma+1)\,\pi_m/\pi^- - K_0] - 2\gamma mS_k^0\mathrm{M}^{-2}(\tau^-)^{-1}\},$$
$$B = \gamma^2(\gamma+1)\,\mathrm{M}^{-2}\,\{(\gamma+1)\,\mathrm{M}_m^{-2}\tau_m/\tau^- + 2\,(mS_k^0)^2\,(\tau^-)^{-1}\},$$
$$C = \gamma^2K_0^2 - [(\gamma+1)\,\pi_m/\pi^- - K_0]^2 - 2\gamma^2(\gamma+1)\,\mathrm{M}^{-2}\,(\tau^-)^{-1},\ K_0 = 1+\gamma\mathrm{M}^{-2}. \tag{7.3}$$

Figure 4 shows the results of the calculation of the coefficient of narrowing of the jet ε_m with a critical efflux of gas ($M_m - 1$, $\gamma = 1.25$) through a side opening mS_k^0 for different starting values of M^- (numbers on the solid lines). For efflux through small side openings ($mS_k^0 \ll 1$) the coefficients ε_m and the flow rate $|\Delta_G|$ (7.2) decrease as the starting velocity (M^-) increases. However, as the size of the openings increases $|\Delta_G|$ and ε_m increase, and to an especially large degree for the gas with large values of M^-. The latter behavior is explained by the increase in the difference $p_\sigma - p_m$ (7.1) owing to the increase in p_σ when mass is removed from the subsonic flow. The coeffficient ε_m reaches a maximum value when the entire flow is diverted through the side openings and $\Delta_G = -1$, $v^+/v^- = 0$ and $\max(p^+/p^-)$ (in Fig. 4 the broken lines are the graphs of p^+/p^-). When the side openings are further increased in size a reverse flow appears in the section S^+ ($v^+ < 0$) and p_σ drops, which reduces ε_m, and in addition $\varepsilon_m \to 0$ as $mS_k^0 \to \infty$. The region of application of the solution (7.3) is bounded by the conditions $M^- < 1$, $M^+ < 1$.

Thus for problems of the hydrodynamics of internal flows with local actions on the flow the hypothesis formulated here about the thermodynamic function σ_p is more universal than previous hypotheses. The consequence of this hypothesis — the additional equation (3.8) — expanded the region of application of the equations of hydrodynamics in integral form for determining the integral characteristics of the flow.

LITERATURE CITED

1. V. Ya. Neiland and N. I. Kukanova, "Investigation of flows with zones of detachment," Obz. Perevody. Ref. BNI TsAGI, No. 129 (1965).
2. I. P. Ginzburg, Applied Hydro- and Gas-Dynamics [in Russian], Leningrad. Gos. Univ., Leningrad (1958).
3. R. E. Sorkin, Theory of Intrachamber Processes in Solid-Fuel Rocket Systems [in Russian], Nauka, Moscow (1983).
4. B. V. Orlov and G. Yu. Mazing, Thermodynamic and Ballistic Fundamentals of the Design of Solid Fuel Rocket Engines [in Russian], Mashinostroenie, Moscow (1979).
5. V. G. Dulov, "Decomposition of an arbitrary discontinuity in the parameters of a gas at a jump in the cross-sectional area," Vestn. Leningrad. Univ., Mat. Mekh. Astron., No. 19 (1958).
6. I. K. Yaushev, "Decomposition of an arbitrary discontinuity in a channel with a jump in the cross-sectional area, " Izv. Sib. Otd. Akad. Nauk SSSR, Ser. Tekh. Nauk., No. 8, Issue 2 (1967).
7. L. I. Sedov, Mechanics of a Continuous Medium [in Russian], Nauka, Moscow (1976), Vol. 2.
8. V. A. Mamaev, G. É. Odishariya, N. I. Semenov, and A. A. Tochilin, Hydrodynamics of Gas-Liquid Mixtures in Pipes [in Russian], Nedra, Moscow (1969).
9. Kh. A. Rakhmatulin, A. Ya. Sagomonyan, A. I. Bunimovich, and N. I. Zverev, Gas Dynamics [in Russian], Vysshaya Shkola, Moscow (1965).
10. V. M. Eroshenko and L. I. Zaichik, Hydrodynamics and Heat and Mass Transfer in Permeable Surfaces [in Russian], Nauka, Moscow (1984).
11. L. I. Sedov, Mechanics of a Continuus Medium [in Russian], Nauka, Moscow (1976), Vol. 1.
12. P. G. Kiselev [ed.], Handbook for Hydraulic Calculations [in Russian], Energiya, Moscow (1972).
13. R. R. Chugaev, Hydraulics [in Russian], Énergoizdat, Leningrad (1982).
14. U. Chester, "Propagation of shock waves in channels with variable cross section," Probl. Mekh., No. 4 (1963).
15. I. E. Ul'yanov, N. N. Krumina, and N. V. Vakar, Design of Air Intake Ducts of Jet Engines [in Russian], Mashinostroenie, Moscow (1979).
16. E. P. Korovin, "Investigation of the impact of gas in a sudden expansion," Tr. SibNIA (1954).